Homework Set 3, S454, February 13, 2012

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Due in one week

For both a Hamming taper

\[ D_{\text{Hamm}}(n) = 0.54 - 0.46 \cos\left(2\pi \frac{n + \frac{1}{2}}{N}\right) \]

where \( n = 0, 1, \ldots, N - 1 \), and a Hanning taper

\[ D_{\text{Hann}}(n) = 0.50 - 0.50 \cos\left(2\pi \frac{n + \frac{1}{2}}{N}\right). \]

1. Compute the scale factor so that

\[ \sum_{n=0}^{N-1} D^2(n) = 1. \]

You may do this either analytically (approximate the sum with an integral) or numerically for \( N = 100 \). **Stat 854 do both.**

2. Take \( N = 100 \), compute and plot both tapers.

3. For the tapers in Part 2, zero pad to \( M = 1000 \) and 1024 and plot the corresponding spectral windows. It will make a “nicer” plots if you clip the lower extremes of the spectral windows at a level of about \( 10^{-12} \times \text{max} \), but choose the clipping point so as not to obscure “real” features. Reminder: a “Rayleigh” is a frequency increment of \( 1/N \), the number of actual data points. Make four plots (use proper frequency units):
   - **A** Linear power and frequency from 0 to 10 Rayleighs;
   - **B** Log Power and linear frequency from 0 to 10 Rayleighs;
   - **C** Log Power and linear frequency from 0 to \( \frac{1}{2} \);
   - **D** Log Power and Log Frequency from 5 Rayleighs to \( \frac{1}{2} \).

How do the two tapers compare in sidelobe level and decay rate? Why?
What differences do you see between zero padding to \( M = 1000 \) and \( M = 1024 \)?