1. **PASTA: Poisson Arrivals See Time Averages.** Consider a continuous time Markov chain observed at the times of a Poisson process with rate $\lambda$. Let $X = \{X(t) : t \geq 0\}$ be a continuous time Markov chain with stationary distribution $\pi$. Let $S_1, S_2, \ldots$ be the event times of a Poisson process with rate $\lambda$. Define $Y_n = X(S_n)$ for $n \geq 1$. Then $Y = \{Y_n : n \geq 1\}$ is a discrete time Markov chain. Show that the stationary distribution of $Y$ is also $\pi$. 

   **Hint:** Show that the transition probabilities for the discrete time chain $Y$ are given by
   \[
   q_{ij} = \int_0^\infty p_{ij}(t) \lambda e^{-\lambda t} dt,
   \]
   where $p_{ij}(t)$ is the $(i,j)$th transition probability function for the $X$ chain.

2. Consider a service system with 2 servers. Customers arrive to the system according to a Poisson process with rate $\lambda$. All service times are independent exponential random variables with rate $\mu$. If a customer arrives to the system when there is at least one server free the customer immediately goes into service and then departs the system once service is completed. Otherwise the customer waits in a queue, which can accomodate an unlimited number of waiting customers. Customers in queue are served in a first-come first-served manner. This system is known as the $M/M/2$ queue. Let $X(t)$ denote the number of customers in the system (in service or in queue) at time $t$.

   (a) Write the state space and infinitesimal generator for the process $\{X(t) : t \geq 0\}$.
   (b) Compute the stationary distribution.
   (c) We say that overtaking occurs when a customer departs the system before another customer who arrived earlier. In steady state, find the probability that an arriving customer overtakes another customer (you may assume that the state of the system at each arrival instant is distributed according to the stationary distribution).