Mathematical Models Of Predator-Prey Systems

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The Mathematics of Invasions in Ecology and Epidemology

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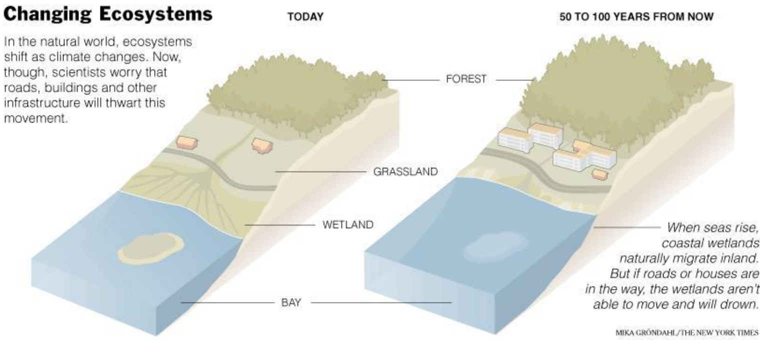
Outline

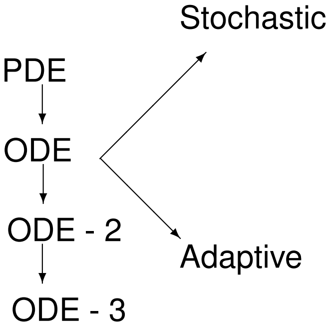
1. Introduction
2. Models
3. Discussion and Conclusions

Introduction

Question: How does changing habitat affect predator-prey dynamics?



Models



Evil PDE's...

PDE Attempt 1: Model prey and predator movement with a reaction diffusion equation and habitat function $h(t, x)$.

$$
\begin{aligned}
N_t &= f(N, P)h(t, x) + D_1 N_{xx} \\
P_t &= kg(N, P)h(t, x) + D_2 P_{xx},
\end{aligned}
$$

where

$$
\begin{aligned}
f(N, P) &= rN\left(1 - \frac{N}{K}\right) - CNP \\
g(N, P) &= BCNP - DP
\end{aligned}
$$

Diffusion Driven Instability ?
Without the diffusion terms the stable equilibrium are

$$(\overline{N}, \overline{P}) = \left(\frac{D}{BC}, \frac{r}{C}\left(1 - \frac{D}{KBC}\right)\right)$$

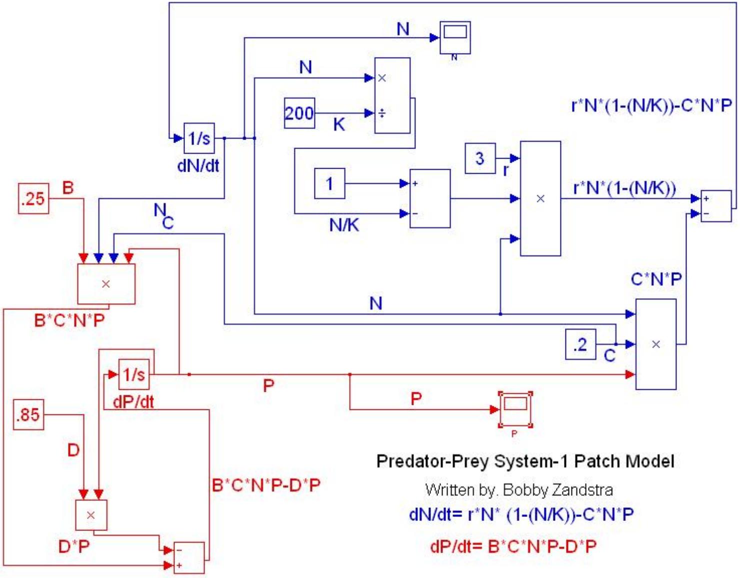
provided $\frac{D}{KBC} < 1$.
When adding in the diffusion terms, there are two new conditions required for diffusion-driven instability. They are $f_N d + g_P > 0$ and $f_N^2 d^2 + 2d(f_N g_P - f_P g_N) + g_P^2 > 0$, where $d = \frac{D_2}{D_1}$. The first condition is a necessary condition, and ...unfortunately we didn't check that one first. The necessary condition is

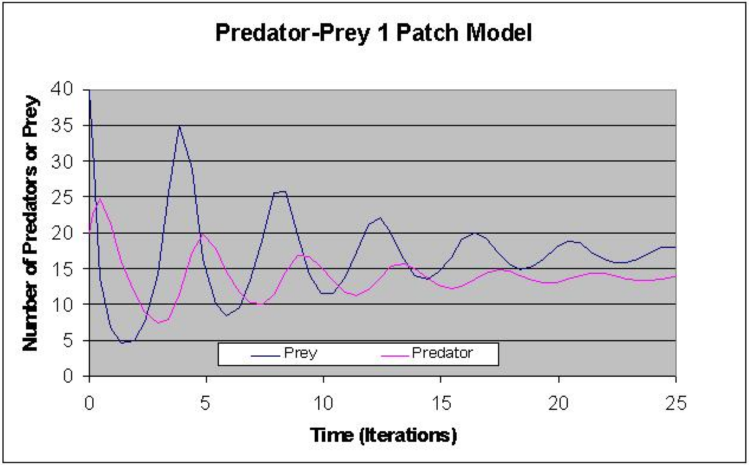
$$-\frac{rDd}{KBC} > 0$$

Model 2:

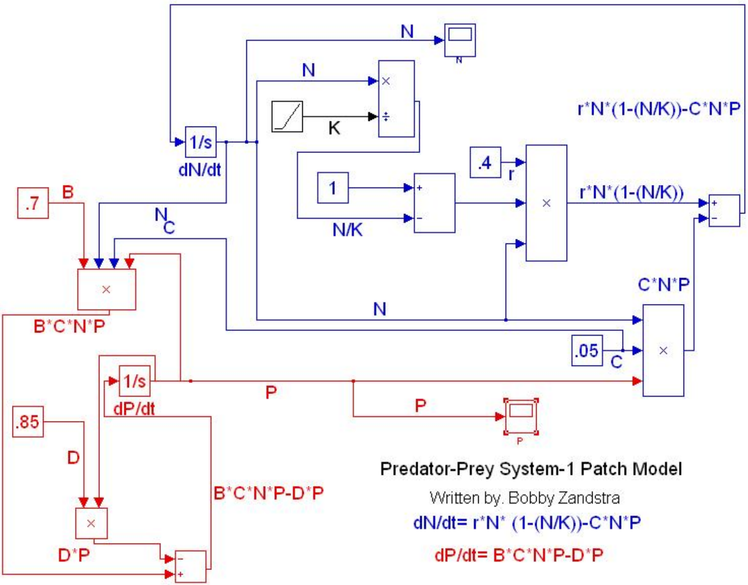
$$\frac{\partial N(x,t)}{\partial t} = rN(x,t)(1 - \frac{N(x,t)}{K(x,t)}) - cN(x,t)P(x,t)$$
$$- D_1 P(x,t)\frac{\partial P(x,t)}{\partial x}$$
$$\frac{\partial P(x,t)}{\partial t} = P(x,t)(BCN(x,t) - D) + D_2 N(x,t)\frac{\partial N(x,t)}{\partial x}$$

ODE - One Patch in Simulink

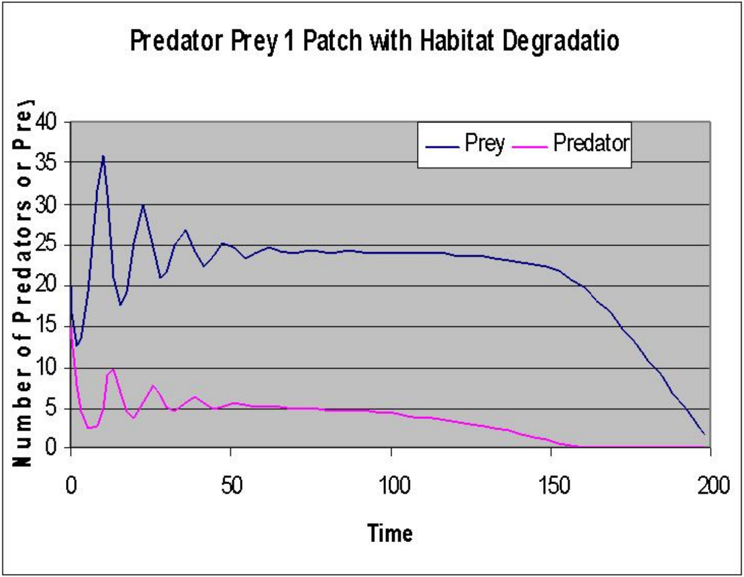




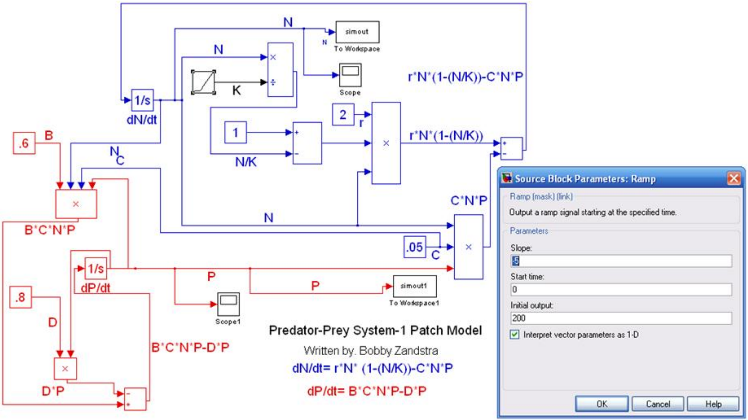
Predator-Prey 1 Patch Model
Number of Predators or Prey
40
35
30
25
20
15
10
5
0
0
5
10
15
20
25
Time (Iterations)
Prey
Predator

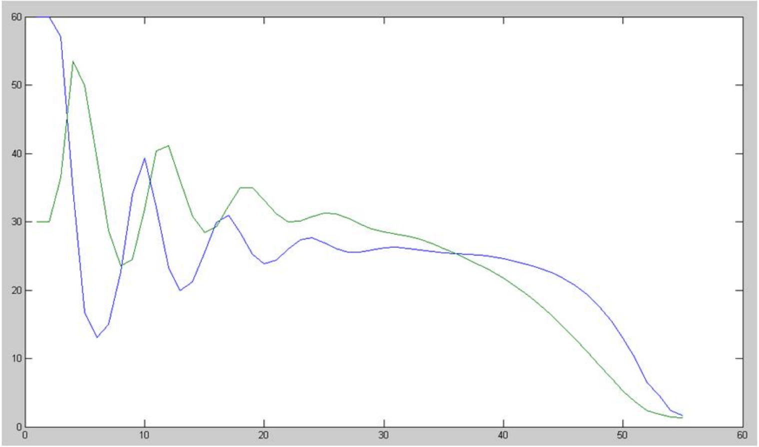


N
N
1/s
dN/dt
K
r*N*(1-(N/K))-C*N*P
.4
r
1
r*N*(1-(N/K))
.7
B
N
C
N/K
C*N*P
B*C*N*P
N
.05
C
1/s
P
dP/dt
P
.85
D
B*C*N*P-D*P
D*P
P
Predator-Prey System-1 Patch Model
Written by. Bobby Zandstra
dN/dt= r*N* (1-(N/K))-C*N*P
dP/dt= B*C*N*P-D*P



Predator Prey 1 Patch with Habitat Degradatio
Number of Predators or Prey
40
35
30
25
20
15
10
5
0
Prey
Predator
0
50
100
150
200
Time


N
simout
To Workspace
N
Scope
K
1/s
dN/dt
r*N*(1-(N/K))-C*N*P
2
r
1
.6
B
N
C
N/K
r*N*(1-(N/K))
C*N*P
B*C*N*P
N
.05
C
1/s
dP/dt
P
P
simout1
To Workspace1
Scope1
.8
D
B*C*N*P-D*P
D*P
Predator-Prey System-1 Patch Model
Written by. Bobby Zandstra
dN/dt= r*N* (1-(N/K))-C*N*P
dP/dt= B*C*N*P-D*P
Source Block Parameters: Ramp
Ramp (mask) (link)
Output a ramp signal starting at the specified time.
Parameters
Slope:
5
Start time:
0
Initial output:
200
Interpret vector parameters as 1-D
OK
Cancel
Help



60
50
40
30
20
10
0
0
10
20
30
40
50
60

Lotka-Volterra with Adaptive Behavior

$$
\begin{aligned}
\frac{dN}{dt} &= r(x)N\left(1-\frac{N}{K}\right) - C(x)NP \\
\frac{dP}{dt} &= BC(x)NP - DP \\
\frac{dx}{dt} &= \nu\left(\frac{d}{dx_m}r(x(t))\left(1-\frac{N}{K}\right) - \frac{d}{dx_m}C(x(t))P\right)
\end{aligned}
$$

where r is intrinsic rate of growth, C is the capture rate of prey, K is the carrying capacity, x is a behaviorial trait that we are tracking (the proportion of time the prey is out searching for food), x_m is a variant of this trait, and ν is a rate of adaptive change .

Explanation of last ODE

We start by defining fitness , W, as

$$W(x_m(t), x(t)) = \frac{\frac{dN}{dt}}{N}$$
$$W(x_m(t), x(t)) = r x_m(t)\left(1 - \frac{N}{K}\right) - C(x_m(t))P$$

Note the the capture rate and growth rate are dependent on the variant trait, x_m. Next we define the change in the trait value as:

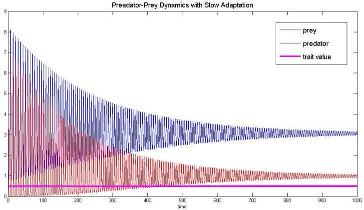
$$\frac{dx}{dt} = v \frac{dW}{dx_m}\bigg]_{x_m(t)=x(t)}.$$

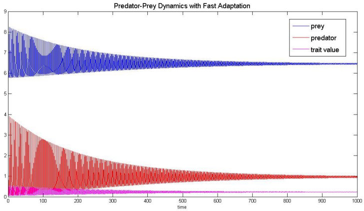
This yields

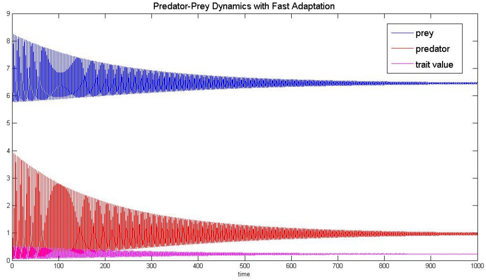
$$\frac{dx}{dt} = v\left(\frac{d}{dx_m} r(x(t))\left(1 - \frac{N}{K}\right) - \frac{d}{dx_m} C(x(t))P\right)$$

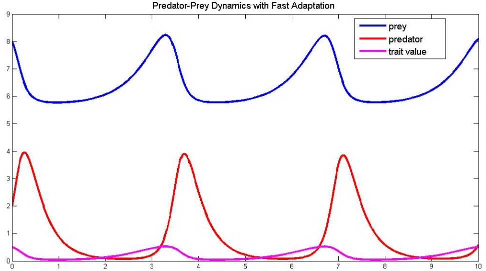
Question of Interest

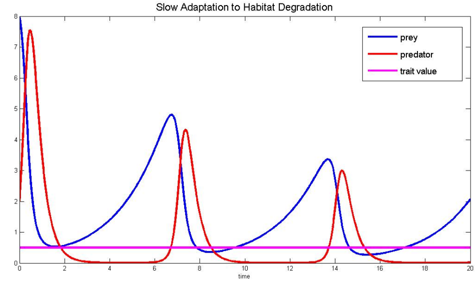
What is the effect of adaptive behavior on predator-prey dynamics?
Numerical Examples:

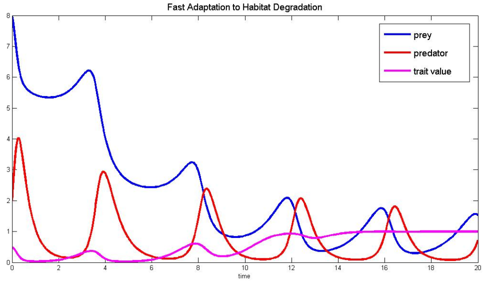





Predator-Prey Dynamics with Fast Adaptation
prey
predator
trait value
time
0 100 200 300 400 500 600 700 800 900 1000
0 1 2 3 4 5 6 7 8 9

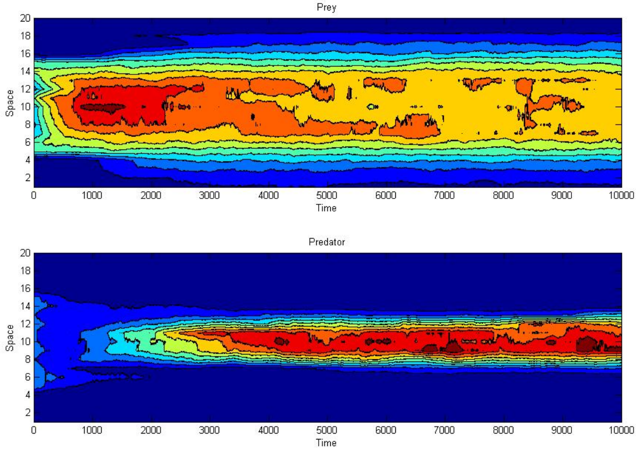

Predator-Prey Dynamics with Fast Adaptation
prey
predator
trait value
0 1 2 3 4 5 6 7 8 9 10
0 1 2 3 4 5 6 7 8 9


Slow Adaptation to Habitat Degradation
prey
predator
trait value
time
0
2
4
6
8
10
12
14
16
18
20
0
1
2
3
4
5
6
7
8


Fast Adaptation to Habitat Degradation
prey
predator
trait value
time
0
2
4
6
8
10
12
14
16
18
20
0
1
2
3
4
5
6
7
8

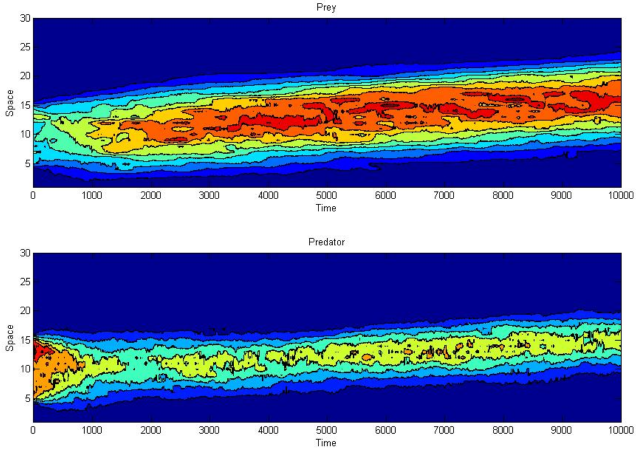
Stochastic Model

Stochastic

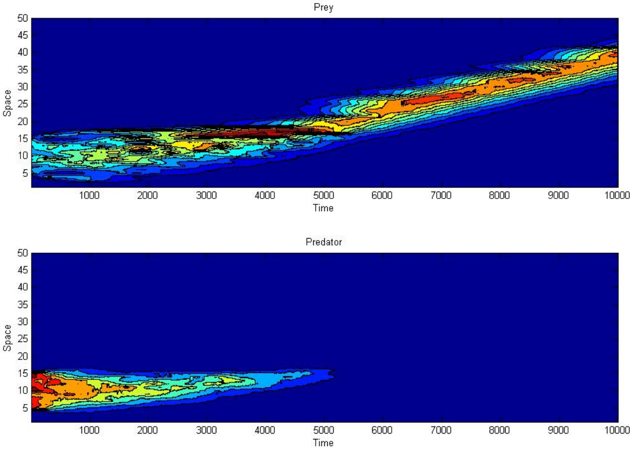


Prey
Space
Time
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
2
4
6
8
10
12
14
16
18
20
Predator
Space
Time
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
2
4
6
8
10
12
14
16
18
20

Stochastic



Stochastic



Prey
50
45
40
35
30
25
20
15
10
5
Space
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
Time
Predator
50
45
40
35
30
25
20
15
10
5
Space
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
Time

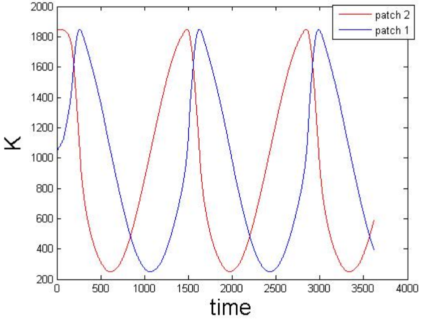
ODE - Two Patch

$$\dot{N}_1 = rN_1(1 - \frac{N_1}{K(t)}) - \gamma_1 N_1(\frac{1+P_1}{1+N_1}) + \gamma_1 N_2(\frac{1+P_2}{1+N_2}) - cP_1N_1$$
$$\dot{P}_1 = bcP_1N_1 - \mu P_1 - \gamma_2(\frac{P_1}{1+P_1N_1}) + \gamma_2(\frac{P_2}{1+P_2N_2})$$
$$\dot{N}_2 = rN_2(1 - \frac{N_2}{\bar{K}(t)}) - \gamma_1 N_2(\frac{1+P_2}{1+N_2}) + \gamma_1 N_1(\frac{1+P_1}{1+N_1}) - cP_2N_2$$
$$\dot{P}_2 = bcP_2N_2 - \mu P_2 - \gamma_2(\frac{P_2}{1+P_2N_2}) + \gamma_2(\frac{P_1}{1+P_1N_1})$$

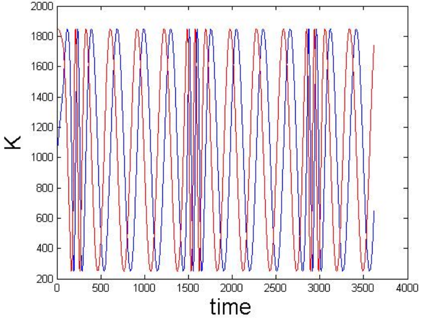
where

$$K(t) = A\cos(t/s_1) + C_1$$
$$\bar{K}(t) = A\sin(t/s_2) + C_2$$

with initial conditions $(N_1(0), P_1(0), 0, 0)$.

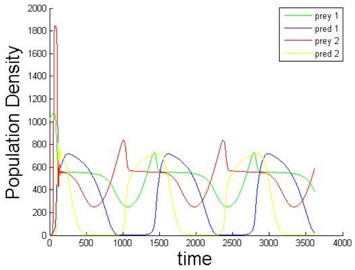


patch 2
patch 1
K
time

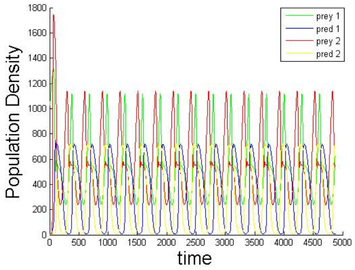


K
time

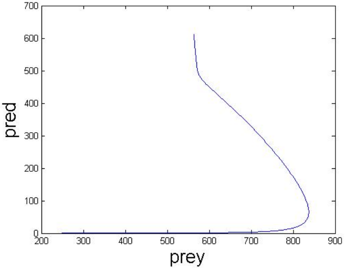
Results



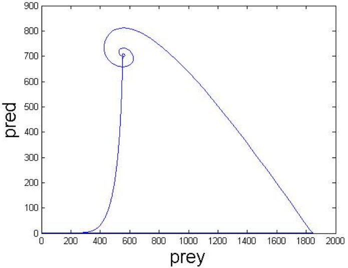
prey 1
pred 1
prey 2
pred 2
Population Density
time
0
500
1000
1500
2000
2500
3000
3500
4000
200
400
600
800
1200
1400
1600
1800



prey 1
pred 1
prey 2
pred 2
Population Density
time
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000
200
400
600
800
1200
1400
1600
1800



pred
prey



pred
prey

Snake vs Bird

ODE - Three Patch

Prey Density = Birth/Death rate -Emigration + Immigration

$$\frac{d}{dt}N_m(t) = r(t)\,N_m - \xi\,N_m\,P_m - \left(\frac{\lambda\,N_m^2}{K_m(t)} + q\,P_m N_m\right)$$

$$+ \sum_{i=1}^{n}\left(\frac{\lambda N_i^2}{K_i} + q\,P_i\,N_m\right)\frac{A_i e^{-\alpha\,dis(m,i)}}{\sum_{j=1}^{n} A_j\,e^{-\alpha\,dis(i,j)}}$$

Predator Density = Birth/Death rate

$$\frac{d}{dt}P_m(t) = b\,\xi\,N_m\,P_m - d\,P_m$$

Description of parameters

Parameter	Interpretation	value
m	Number of Patches	3
N_m	Density of Prey	
P_m	Density of Predator	
A_i	Area/Qulity of Patch $i (\in \{1, .., m\})$	20/5/15
$K_i(t)$	Fluctuating carrying capacity dependent on area A_i and time	$\begin{bmatrix} 20 + 3\,\frac{\sin(t)}{t+1} \\ 5 - 2\,\frac{\sin(t)}{t+1} \\ 3 + \frac{\cos(t)}{t+1} \end{bmatrix}$
dis	Distance/Difficulty between patch i and j	
b	Predator birth coefficient	1.5
ξ	Prey catch rate	1
λ	Prey dispersal rate	5
α	Distance coefficient	1
q	**SCARE** factor	.3
γ	Leaving coefficient	.5

Table: Description of parameters and their values

Predator gets smart!

Predator Density = Birth/Death rate -Emigration + Immigration

$$\frac{d}{dt}P_m(t) = b\,\xi\,N_m\,P_m - d\,P_m - \frac{P_m(t)\gamma}{1+N_m(t)} + \sum_{i=1}^{n}\left(\frac{\gamma\,P_i}{(1+N_i)}\right)\frac{\left(\frac{N_i}{N_m}\right)}{\left(\sum_{j=1}^{n}\frac{N_j}{N_i}\right)}$$

Thank you for listening!
Thank you to the organizers, BIRS and other participants