$V$ finite dim. vector space over a field $F$

$G$ acting linearly on $V$, then there is an induced action on $F^G(V)$

$F^G(V) = \{ f \in F^V \mid gf = f, \forall g \in G \}$

**Definition:** A subset $X \subseteq F^G(V)$ is separating if $\forall x, y \in V$ we have

if $f(x) = f(y)$ $\forall f \in X$, then $f(x) = f(y)$ $\forall f \in F^G(V)$

**Theorem (Derkx - Kemper, 2002):** All invariant rings have finite separating sets.
(Petersen - Kemper, 2002)
- Noether bound holds for separating invariants in all characteristics

- (Oraisma, Kemper, Wehlau) 2006
  Weyl's polarization theorem holds for separating invariants in all characteristics

- (Domokos, 2007)
  Further efficiency results on sep. invariants for decomposable representations.
modular separating and generating invariants

\[ \text{char } F = \rho \]

G is 2l/p has \( \rho^2 \) indecomposable representations

\[ V_1 = \begin{pmatrix}
1 & 0 & 0 & \cdots & 0 \\
1 & 1 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & 1 \\
0 & 0 & 0 & \cdots & 1
\end{pmatrix} \]

\[ 1 \leq \rho \leq p^r \]

FC\( V_3 \) \( Z_p \), FC\( V_3 \) \( Z_p \) \( \rho \) (Dickson 1913)
FC\( V_2 \), FC\( V_3 \) \( Z_p \) (Shank 1998)
FC\( mV_2 \) \( Z_p \) (Campbell-Hughes, 1997)
FC\( 2V_3 \) \( Z_p \) (Campbell-Fodden-Wehlau, 2006)
FC\( V_2 + V_3 \) \( Z_p \) (Shank-Wehlau, 2002)
FC\( V_1 \) \( Z_p \) is \( G \), FC\( W \) \( Z_p \), where each index
summed in \( W \) has dimension at most 4, (Wehlau, 2009)
(Fleischmann, M.S., Shank, Woodcock, 2006)

Exact degree required to generate $FCU\mathbb{Z}_{2}$ for any $V$ is given,

\[ \mathbb{Z}/p^r \to \]

i) $FCU_{p+1} \mathbb{Z}_{2}^r$ (Shank-Wehlau, 2005)

ii) $FCU_{1} \mathbb{Z}_{2}^r$ (Neusel, Sezer, 2008)

An infinite generating set is given; norms, transfers, and invariants up to some degree.

(Symonds, 2008) $FCU_{1} \mathbb{Z}_{2}^6$ is generated by invariants of degree at most\[ \dim V - (|G| - 1) \]

$V, G$ arbitrary.
separating invariants

G any p-group.
e_1, e_2, ..., e_n be a basis for V^G
x_1, x_2, ..., x_n be the corresponding elements in V^G

(Netzel, Mis) subalgebra of FCWJ^G generated
by V(x_i) (i \leq n) and I is separating.

I = \bigcap_{H \text{ maximal}} \text{Tr}_H^G

An Approach:
we want to find a separating set for I.
Assume
\[ T: V \rightarrow W \text{ G-equivariant surjection} \]
and T is a separating set for W.

FCWJ \rightarrow FCVJ.

say we want to separate u_1, u_2 \in V.
if \[ \Pi(u_1) \text{ and } \Pi(u_2) \] are in different orbits,
a polynomial in T separates u_1, u_2.
Theorem: Let $S$ be a separating set for $V_{n-1}$. Then $S$ together with $W(x_n)$ and $\operatorname{Tr}(x_n x_i^r)$, $1 \leq i \leq n-2$, is a separating set for $V_n$.

How does this generalize to $\mathbb{Z}/p^n$?

$s \leq n-2$ with $p^{k-1}+1 \leq n-i \leq p^k$

$G = \mathbb{Z}/p^n$, $G_k = \mathbb{Z}/p^{n-k}$

$H(\cdot) = \operatorname{Tr}_{G_k} G (W_{G_k}(x_n) \prod_{0 \leq j \leq k-1} (W_{G_k}(x_{i+j}))^{-1})$

Lemma: $H(\cdot) = W_{G_k}(x_n) x_i^{p^{r-k}(p-1)} + f$ modulo $(x_{i-1}, \ldots, x_i)$ where $f \in F[C_i, x_{i+1}, \ldots, x_n]$.
Theorem: \(1 \leq n \leq p\)

Let \(s \subseteq F_{n-1}\) be a separating set for \(V_{n-1}\), then \(s\) together with \(V_{x_{2n}}\) and \(H(C_i) i \leq i \leq n-2\) is a separating set for \(V_n\).