

Errata to Complete Caps in Projective Space which are Disjoint from a Subspace of Codimension Two

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The published version of *Complete Caps in Projective Space which are Disjoint from a Subspace of Codimension Two* contains an error in the proof of Theorem 4.1. This error affects Proposition 7.1 and Proposition 8.1.

The correct statement and proof of Theorem 4.1 are as follows:

Theorem 4.1 *Suppose that A and B are such that Equation 1.3 is satisfied for all $i = 1, 2, \dots, 2^{n-r-1}$. Suppose $0 < t + u < 2^{n-r-1}$, that $\overline{t \neq u}$, and that either $\overline{A_f}$ or $\overline{B_f}$ is non-periodic. Further suppose that $\widehat{C} \setminus C \subseteq \cup_{i=1}^{2^{n-r-1}} (A(i) + B(i))$. Then S is a complete cap in Σ .*

Proof By construction, S is a cap in Σ . Also Equation 1.3 guarantees that every point of A' and every point of B' lies on at least one secant of S through a point of C . Thus we need to show that every point of H_∞ and every point of $H_C \setminus C$ lies on a secant of S .

Without loss of generality suppose that $\overline{A_f}$ is non-periodic. Consider the cap $\overline{S_1}$ in $\overline{\Sigma}$ given by $\overline{S_1} = \overline{A_f} \sqcup \overline{B_{ne}} \sqcup \{C\}$. Applying Theorem 3.5 we see that for every point $\overline{c_k}$ of $\overline{H_C}$ different from \overline{C} there exists $i \leq t$ and $j \geq t+1$ such that $\overline{a_i} + \overline{b_j} = \overline{c_k}$. Since the coset $H_A(i)$ is entirely contained in S , and $H_B(j) \cap S \neq \emptyset$, this means that every point of the coset $H_C(k) = H_A(i) + H_B(j)$ lies on a secant to S .

By assumption, every point of $\widehat{C} \setminus C$ is contained in $A + B$.

Hence it only remains to prove that every point of H_∞ lies on at least one secant to S . Note that every point of F lies in $H_A(1) \oplus H_A(1)$ (or in $H_B(1) \oplus H_B(1)$ if $t = 0$). Thus we consider a point $z \in H_\infty \setminus F$ and write \overline{z} for the point of $\overline{\Sigma}$ corresponding to the coset of F generated by z .

As above, if $\overline{z} \in \overline{A_f} + \overline{A_{ne}}$ or $\overline{z} \in \overline{B_f} + \overline{B_{ne}}$ then z lies on a secant line to S . Thus we assume, by way of contradiction, that $\overline{z} + \overline{A_f} \subset \overline{A_e}$ and $\overline{z} + \overline{B_f} \subset \overline{B_e}$. The first inclusion implies that $t \leq u$ while the second implies that $u \leq t$. Thus $t = u$ contradicting our hypothesis.

□

Here are the corrected versions of Remark 4.3 and Remark 4.4.

Remark 4.3 Note that, if t or u is odd, then $\overline{A_f}$ or $\overline{B_f}$ respectively is non-periodic. Thus for a fixed value of $t + u$ with $1 \leq t + u \leq 2^{n-r-1} - 1$ we may take $t = 1$, for example, to arrange that $\overline{A_f}$ is non-periodic.

Furthermore, if $t + u \neq 2$ we may simultaneously arrange that $t \neq u$.

Remark 4.4 There do exist complete caps for which $t + u = 0$ (and thus $\overline{A_f} = \overline{B_f} = \emptyset$). See Example 5.4.

The condition that $t \neq u$ is also sufficient but not necessary.

The corrected version of paragraph 4 of Section 5, as well as two new paragraphs which should follow paragraph 4 are give below:

Then $|S| = (2^r - 1) + 2s + 2^r(2^{n-r-1} - s) = 2^{n-1} + 2^r - 1 - (2^r - 2)s$. By Remark 4.2, s cannot equal 0. Furthermore by Theorem 4.1 (and Remark 4.3) there do exist complete caps of this form for all $s = 1, 2, \dots, 2^{n-r-1} - 3$ and

$s = 2^{n-r-1} - 1$. Thus we find complete caps of this form of all cardinalities: $2^{n-r} + k(2^r - 2) + 1$ for $k = 2, 3, \dots, 2^{n-r-1} - 2$ and $k = 2^{n-r-1}$ where $2 \leq r \leq n - 2$.

Next we consider the case $s = 2^{n-r-1} - 2$. Then $u + v = 2$ and either $u = v$ or else both $\overline{A_f}$ and $\overline{B_f}$ are periodic. Reordering the cosets we may suppose $|H_A(i)| = |H_B(i)| = 1$ for $i = 3, 4, \dots, 2^{n-r-1}$. Write $\{\alpha_i\} = H_A(i)$ and $\{\beta_i\} = H_B(i)$ for $i = 3, 4, \dots, 2^{n-r-1}$. We consider first the case that $t = u = 1$. Examining the proof of Theorem 4.1 we see that every point of $\mathbb{P}\mathbb{G}(n, 2)$ lies on a secant to S except possibly the 2^r points in the coset $H_A(1) + H_A(2) = H_B(1) + H_B(2) = \{z_1, z_2, \dots, z_{2^r}\} \subset H_\infty$. These points z_k can only lie on secants of the form $\alpha_i + \alpha_j = z_k$ or $\beta_i + \beta_j = z_k$. Since $\beta_i + \beta_j = \alpha_i + \alpha_j$ we may restrict our attention to the α_i . Thus we see that S is complete if and only if for every $k = 1, 2, \dots, 2^r$ there exists $3 \leq i(k) < j(k) \leq 2^{n-r-1}$ such that $\alpha_{i(k)} + \alpha_{j(k)} = z_k$. Since the α_i all lie in different cosets, it is clear that $\alpha_{i(1)}, \alpha_{j(1)}, \dots, \alpha_{i(2^r)}, \alpha_{j(2^r)}$ must be distinct. Thus we must have $2(2^r) \leq 2^{n-r-1} - 2$ or equivalently $2r + 3 \leq n$.

The case $(t, u) = (2, 0)$ (or $(0, 2)$) is entirely similar to the above case. The only difference being that the 2^r points which may not lie on secants to S form a coset in H_C instead of in H_∞ . In summary we see that there exists a complete cap S with $|\widehat{C} \setminus C| = 1$ and with $|S| = 2^{n-r} + 2^{r+1} + 2^r - 5$ (and $s = 2^{n-r-1} - 2$) if and only if $n \geq 2r + 3$.

Here is the corrected version of Proposition 7.1:

Proposition 7.1 *Let S be a complete cap in $\mathbb{P}\mathbb{G}(n, 2)$ with $n \geq 4$. Further suppose that S meets a hyperplane K_C of $\mathbb{P}\mathbb{G}(n, 2)$ in exactly 3 points. Then $|S|$ is one of the numbers: $2^{n-2} + 5, 2^{n-2} + 9, 2^{n-2} + 11, 2^{n-2} + 13, \dots, 2^{n-1} + 1$ or, if $n \geq 7$, $|S|$ may also be $2^{n-2} + 3$ or $2^{n-2} + 7$. Moreover complete caps of all these sizes exist for all corresponding values of n .*

Here is the corrected version of Proposition 8.1

Proposition 8.1 *Let S be a complete cap in $\mathbb{P}\mathbb{G}(n, 2)$ with $n \geq 5$. Further suppose that S meets a hyperplane K_C of $\mathbb{P}\mathbb{G}(n, 2)$ in exactly 4 points. Then $2^{n-2} + 8 \leq |S| \leq 2^{n-1} - 2$ or else $|S| = 2^{n-1} + 1$.*

Moreover except for $n = 6$ for sizes 29 and 30, complete caps of all these sizes and this structure exist for all $n \geq 5$.

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I also thank Alexander Davydov for suggesting I clarify the statement of Proposition 8.1 (by adding and this structure) and for pointing out an error on page 355 line -11 (=last sentence on page 7 of the corrected version). The valid values for k are $k = \span style="border: 1px solid black; padding: 0 2px;">2, 4, 5, \dots, 2^{n-r-1}$ where $2 \leq r \leq n - 2$.

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