

# Queen's Algebraic Geometry

## — Seminar —

### RESOLUTIONS OF RINGS OF INVARIANTS

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#### Abstract

Let  $G$  be the cyclic group of order  $n$  and suppose  $\mathbf{F}$  is a field containing a primitive  $n^{\text{th}}$  root of unity. We denote by  $W_b$  the one dimensional representation of  $G$  associated to the character  $-b$  where  $1 \leq b \leq n$ . We consider the ring of invariants  $\mathbf{F}[W]^G$  of the three dimensional representation  $W = W_b \oplus W_c \oplus W_d$  of  $G$  where  $G \subset \text{SL}(W)$ . We describe minimal generators and relations for this ring of invariants and prove that the lead terms of the relations are quadratic with respect to a carefully chosen term order. With this term order these minimal generators for the relations form a Gröbner basis and the lead terms have a surprisingly simple combinatorial structure. Exploiting this structure, we describe the graded Betti numbers for a minimal free resolution of these rings of invariants. The case where  $W = W_b \oplus W_c$  is any two dimensional representation of  $G$  is also handled.

Monday 2 April 2012  
15:30 – 16:30  
319 Jeffery Hall