

The number of genus 2 covers of an elliptic curve

Ernst Kani

Abstract. The main aim of this paper is to determine the number $c_{N,D}$ of genus 2 covers of an elliptic curve E of fixed degree $N \geq 1$ and fixed discriminant divisor $D \in \text{Div}(E)$. In the case that D is reduced, this formula is due to Dijkgraaf.

The basic technique here for determining $c_{N,D}$ is to exploit the geometry of a certain compactification $\mathcal{C} = \mathcal{C}_{E,N}$ of the universal genus 2 curve over the *Hurwitz space* $H_{E,N}$ which classifies (normalized) genus 2 covers of degree N of E . Thus, a secondary aim of this paper is to study the geometry of \mathcal{C} . For example, the structure of its degenerate fibres is determined, and this yields formulae for the numerical invariants of \mathcal{C} which are also of independent interest.