Numerical Methods

1. Why we need them.

Unfortunately it's not always possible to find an explicit antiderivative of a function. The most famous examples are $\sin(x)/x$ and e^{x^2}. Neither of these functions have an antiderivative that can be written in terms of any function you know.

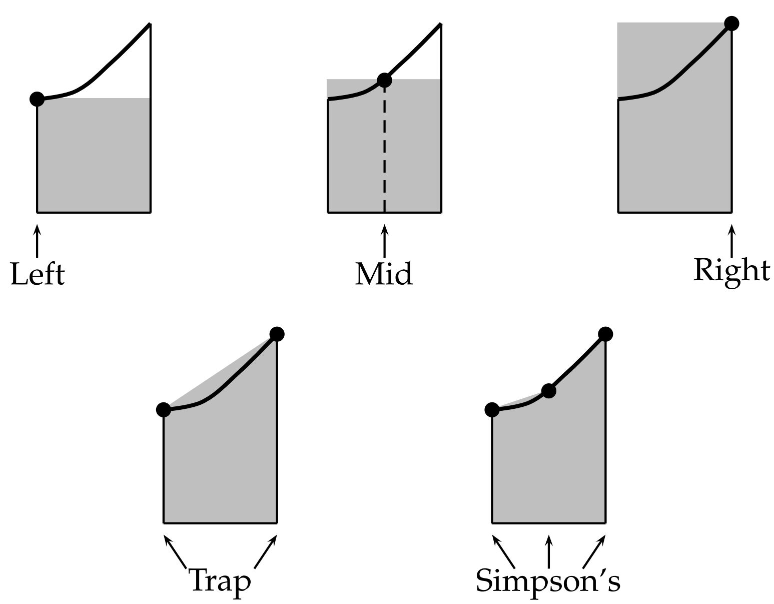
This kind of thing happens more often than you might expect. For instance, at our current level of mathematical technology we don't really know how to solve the equations of fluid flow, and so almost all of the calculations of this type: air flow over an airplane wing, dynamics of the interior of the sun, groundwater flow, have to be done using numerical algorithms to approximate the solution. The computations usually also involve massive machine power.

2. The methods we'll use.

The basic idea behind any numerical method is to break the problem into pieces, approximate the solution on each piece, and then add up the approximations.

For instance, to approximate the area under the graph of $y = f(x)$ on an interval $[a, b]$, we'll break the interval up into a number of pieces, approximate the graph on each piece with a simple geometric figure, and then add up the areas of the geometric figures.

The geometric figures we'll use are rectangles, trapezoids, and, the most complicated, parabolas. The method with parabolas is called Simpson's rule, the method with trapezoids the trapezoid rule, and the method with rectangles left, right, or midpoint rule, depending on how we pick the height of the rectangles.

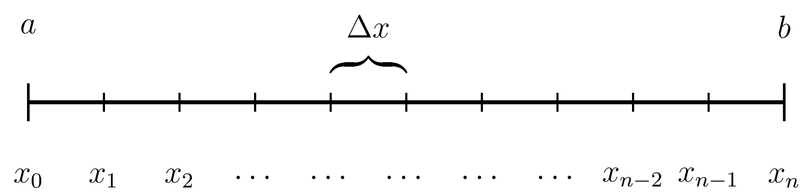


3. Notation.

There are a bunch of symbols that come along with the formulas for the numerical methods, which are nothing more than standardized notation for the process of chopping up the interval $[a, b]$ and describing the important numbers you get by doing that. It's the same notation as for Riemann sums.

If we split the interval $[a, b]$ evenly into n pieces, then the symbols $\Delta x, x_0, \dots, x_n$ mean:

> Δx – the width of each piece.
>
> x_0 – the number a.
>
> x_n – the number b.
>
> $x_1, \dots, x_{n-1}$ – the numbers in between a and b, when the interval is evenly divided into n pieces.



It's sometimes also useful to use the symbols $y_0, \dots, y_n$, where y_i means the height of the graph $y = f(x)$ over x_i. That is, $y_i = f(x_i)$.

This handout can (soon) be found at

http://www.mast.queensu.ca/∼mikeroth/calculus/calculus.html

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