Pouliot Address.
By Peter Taylor, Queen's University.
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The Structure of a Mathematics Curriculum.

It is an honour to receive the 2006 Pouliot Award for contributions to Mathematics Education. The many past winners of this award have made a great impact on mathematics education in Canada, and I'm proud to be among them. In my talk I will review the main concerns and projects of 40 years of work.

We are privileged to study an extraordinary subject of enormous beauty and power. Pure mathematicians and physicists know this well, but even in my work in mathematical biology I am often blown away by the elegance of many of the results.

But this is a well kept secret. I assure you that hardly any high school students, and precious few university students, know about this. We are diligent in keeping this a secret from all but the privileged few.

That's a tragedy—a tragedy for the subject because it's so awesome, a tragedy for our students because they miss something they need at a deep level, and it is bad news for the future of the world.

The book I am reading right now is called *Heat*: *how to stop the planet from burning.* Malcolm Griffin lent it to me and told me it was the climate change book not to miss. The author, George Monbiot, declares at the beginning: *Nothing here is as it seems. The research for this book has involved me in a long series of surprises. What I have sought to do throughout the text is to start from first principles, to believe nothing until it is demonstrated, to junk any technology, however pleasing it may be, which does not work*. Well that's what mathematics does, and that's one reason the world needs it so badly.

But not enough kids are studying engineering, physical science and math. Most find mathematics boring and irrelevant. Many are intimidated by it and find that it fills them with anxiety. Certainly almost no one finds school mathematics exciting or engaging. We need to try to understand why this is the case.

On many occasions over the past 8 years I have worked with the Ministry of Education in Ontario in the process of writing the policy documents for the senior school math curriculum. It's a challenging process as we must navigate around a number of different views on content and emphasis. The final product is pedestrian and might be compared to a manual for assembling a vacuum cleaner—something you do need to study at one or two points in your life, but otherwise it just stays on the shelf. Parts of it are devoted to a discussion of the importance of process elements—engaging the student in investigation and inquiry, but most of it is a list of the technical skills we want our students to obtain at that level. In designing a curriculum, that kind of information is of course crucial.

So it's an important document but it's a policy document, not a curriculum. It was never intended to be used as a curriculum. The problem is that that's precisely what it has been used for. To be more precise, the problem is that it has been used as the structural framework for the curriculum.

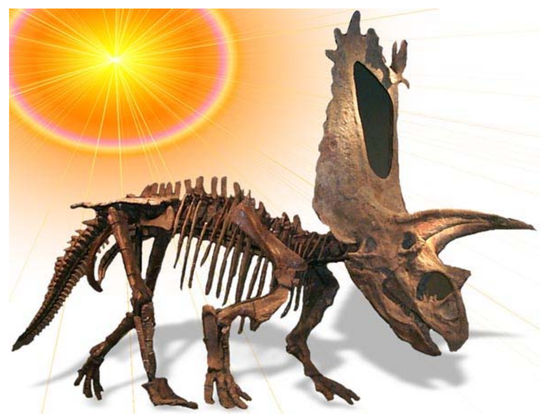
The problem then is one of structure. The structure that serves the policy document is not meant for the curriculum. If so used it bestows the kiss of death.

The reasons for this are complex and no doubt partly known to most readers. It's deemed important (even essential) to cover all the material prescribed in the policy document and the simplest way to ensure this is to follow its logical structure. Even more, it typically appears impossible to cover everything *unless* it is closely followed. Indeed there is so much in the document, that following it is guaranteed to occupy your full class time. I know this because teachers frequently come to me and say that they like my investigative problems but there's never enough time to work with them properly.

Well since investigation and engagement are key components of learning, a natural response is to try to cut the curriculum back a bit to make room for more of this. But this is inevitably problematic as we find that nothing really should be cut. Everything seems equally important.

The irony is that there *is* a way to cover the policy document *and* do the investigative examples, but it requires letting go of the structure of the policy document. It requires putting the entire document aside and basing the curriculum on an entirely different structure.

I find the following analogy helpful. Suppose you wanted to build an animal that could fly and you were given as a starting point the structure at the right. Let's call this the *advanced functions* animal. Others are possible—we could have had the geometry or discrete math animal. But this advanced functions animal, which is the heaviest of all the beasts, is the one deemed most relevant to the needs of the world; its job in life is to forage and dominate the environment and kill and eat the other less fortunate beasts. [You can see where math anxiety comes from.]



So that's your starting point. You see right away that the structure is too heavy to ever get airborne, and must be cut down. But an interesting and embarrassing thing happens. You discover that's almost impossible to cut anything out. The harder you try, the more you see that everything is actually needed. Protection requires big horns, and feeding requires big jaws. You've got to eat after all and the less fortunate beasts are not so easy to crush. So cutting down is not ultimately going to get you there.

[In 1992 I was at an NSF reform calculus conference in San Antonio and at one point, to make some room for the "new" problems, Andy Gleason sent us all off in small groups each to take a portion of the calculus curriculum and see what we could jettison. Guess what?—everything was deemed to be necessary.]

What you need is a completely different starting point. A different structure. Something more like the one on the right. I can imagine building wings on this guy (in fact that's exactly what evolution decided to do). I can imagine using this guy as the basis of a curriculum.

A traditional curriculum is based on a partition of mathematics into different fields (geometry, algebra, functions, probability, discrete math) and these sub-fields are typically built up logically and sequentially. But that's not how mathematicians work. For them, the starting point is a problem (which often belongs to a subfield) and the techniques they use and develop are the ones that have a chance of serving the problem. That's a more *active* design and more intense and engaging. It also happens to fit rather well the operational mode of today's young people.

For me, a curriculum is a network (a better term than "sequence") of investigations, exhibits, works of art, demonstrations, playgrounds, microworlds, grains of sand, whatever you want to call them, which uncover the secrets of the material and provide opportunities for conceptual growth and technical mastery. The surprise is that a good network can actually cover all the significant material in the policy document at the same time as it implements its many process-oriented objectives. In fact the coverage is even better because it's in a natural context.

However the curriculum we construct in this way will often, particularly at the beginning, seems quite disjoint from the policy document and we worry that we won't cover it all and we find this difficult to handle. A big job of "letting go" is needed here, of trusting teachers and students, of trusting the material itself to work its magic. [By the way, there's a lot to be gained from giving the teachers a strong message that we trust them.] The paradoxical truth is that *the only way to be true to the policy document is to let it go.* And I am convinced (after many years of trying different things at different levels and giving problems to different kinds of students and observing their struggles and their successes and failures) that this approach can cover the ground every bit as well as five loaves and two well-chosen small fishes can feed a multitude. But it is a long-term approach rather than short-term. It accomplishes its objective over many years.

As an illustration of this issue, a recent dilemma in the design of the senior math curriculum in Ontario, we had the problem of what to put in the final course for university-bound science majors. Should it be geometry, discrete math, vectors, or calculus, or maybe we could fit two of the four? But why not have all of them and none of them, and call it something like optimization (to take an idea of Ed Barbeau) and just solve a bunch of wonderful max-min problems? I bet we'd cover some important ideas in all four of those areas and learn a lot more real mathematics while we were at it. One could hardly imagine such a course being accepted under the current curriculum philosophy. But if it was, who would be harmed?

Well, maybe that's our work cut out for us over the next 10 years.

Notes:
A number of examples of the investigative problems I work with are found on my website at:
http://www.mast.queensu.ca/~peter/investigations/index.html
In the Pouliot talk itself, I highlighted the *Tire pressure* model and *Throwing balls into boxes.*

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1. National Oceanic & Atmospheric Administration (NOAA), NOAA Central Library.
http://www.history.noaa.gov/art/noaa_gallery/nart0007.html

2. National Science Foundation Directorate for Biological Sciences.
http://www.nsf.gov/news/news_summ.jsp?cntn_id=104498&org=EF